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Study of the double-beta-decay of $100 < A \leq 150$ nuclei within the QRPA, RQRPA and the SRQRPA formalisms

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Abstract. We have used a self-consistent version of the BCS + RQRPA method for a systematic study of the two-neutrino double-beta-decay of nuclei with $100 < A \leq 150$. The comparison with other approaches, namely the QRPA and the RQRPA, shows that inclusion of the quasiparticle correlations at the BCS level reduces the ground-state correlations in the particle-particle channel of the proton-neutron interaction. This in turn results in a systematic reduction of the double-beta-decay matrix elements. The effect of the extension of the formalism on the Ikeda sum rule has been discussed.

PACS. 21.60.Jz Hartree-Fock and random-phase approximations – 23.40.Bw Weak-interaction and lepton (including neutrino) aspects – 23.40.Hc Relation with nuclear matrix elements and nuclear structure – 27.60.+j $90 \leq A \leq 149$

1 Introduction

The Random Phase Approximation (RPA), since its origin in the late fifties and early sixties (see [1] and references therein), has become a very powerful tool for studying the nuclear structure. In particular, the quasiparticle version of the theory (the Quasiparticle Random Phase Approximation —QRPA) has been successfully applied to the nuclei far from the closed shells, and consequently extended as the proton-neutron QRPA (*pn*QRPA) to the description of charge-changing transitions in nuclei [2–9]. Among those transitions, the double-beta-decay draws very much attention, since its proper description at the nuclear level allows (and is necessary) to understand such phenomena as the origin and value of the neutrino mass, the existence of right-handed gauge bosons and other fundamentals of the Standard Model [10,11].

The main drawback in the formulation of the QRPA theory, however, is the violation of the Pauli exclusion principle, connected with the usage of bosonic commutation relations for the QRPA phonon operators, that are in fact collective pairs of fermions. To overcome this shortcoming of the QRPA framework, the renormalization technique has been proposed [12] and extended to include proton-neutron pairing [13]. This approach has been based on the early works by Rowe [1], Hara [14], Ikeda [15] and Schuck and Ethofer [16] in the context of RPA and QRPA. The main goal of the method, called in the literature the renormalized QRPA (RQRPA), is to take into account additional one-quasiparticle scattering terms in the commutation relations by a self-iteration of the QRPA equation.

Recently, we have developed and presented [17] an extension to the RQRPA formalism, that tries to solve the problem of non-vanishing quasiparticle content of the ground state that in turn introduces some inconsistency between RQRPA and the BCS approach. Our method, called the self-consistent RQRPA (SRQRPA), is based on the reformulation of the BCS equations [18] and further reiteration of the BCS + RQRPA calculation scheme. This formalism has been successfully applied to the two-neutrino double-beta-decay of medium-heavy nuclei [17].

This work contains our studies of the double-beta-decay in the $100 < A \leq 150$ mass region. We calculate the double Gamow-Teller matrix elements, using the QRPA, RQRPA and the SRQRPA formalisms. We compare the dependence of the matrix elements on the strength of the particle-particle force, obtained in these approaches and discuss the results. We also quote the corresponding half-life times and compare them with the experimental data. The question of the Ikeda sum rule conservation is raised.

2 Calculation procedure

Since the formalism of the SRQRPA has been presented in detail in our previous publications [17], here we present only the basics of the theory. Since we are interested in the

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charge-changing transitions only, from now on we restrict ourselves to the proton-neutron version of the theory. In the RQRPA and SRQRPA, one introduces the so-called renormalization matrix D_{pn}, defined by the expectation value of the commutator of the angular-momentum coupled bi-quasifermion operators:

$$D_{pn} \equiv \left\langle 0 \left| \left[A_{(pn)J^\pi M}, A^\dagger_{(pn)J^\pi M} \right] \right| 0 \right\rangle = (1 - n_p - n_n), \quad (1)$$

where n_p and n_n are the RPA ground-state quasiparticle densities:

$$n_p \equiv \hat{j}_p^{-1} \left\langle 0 \left| \left[a_p^\dagger \tilde{a}_p \right]_{00} \right| 0 \right\rangle,$$
$$n_n \equiv \hat{j}_n^{-1} \left\langle 0 \left| \left[a_n^\dagger \tilde{a}_n \right]_{00} \right| 0 \right\rangle. \quad (2)$$

With the help of the D_{pn}-matrix, one can introduce the renormalized angular-momentum coupled two-quasiparticle creation operators [19]

$$\mathcal{A}^\dagger_{(pn)J^\pi M} \equiv D_{pn}^{-1/2} \left[a_p^\dagger a_n^\dagger \right]_{J^\pi M}, \quad (3)$$

that behave as bosons, as far as the ground-state expectation value of their commutator is concerned. Assuming the harmonicity of the nuclear motion, the excited-state creation phonon operators can be written as [1,20]

$$Q^{m\dagger}_{J^\pi M} = \sum_{pn} \left[\mathcal{X}^m_{(pn)J^\pi} \mathcal{A}^\dagger_{(pn)J^\pi M} - \mathcal{Y}^m_{(pn)J^\pi} \tilde{\mathcal{A}}_{(pn)J^\pi M} \right]. \quad (4)$$

Using, *e.g.*, the equation of motion (EOM) method [1], one gets the RQRPA equations in the usual form, with $\Omega^m_{J^\pi} \equiv E_{m,J^\pi} - E_{\mathrm{gs}}$ being the energy of the QRPA phonon:

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B} & \mathcal{A} \end{pmatrix}_{J^\pi} \begin{pmatrix} \mathcal{X}^m \\ \mathcal{Y}^m \end{pmatrix}_{J^\pi} = \Omega^m_{J^\pi} \begin{pmatrix} \mathcal{X}^m \\ -\mathcal{Y}^m \end{pmatrix}_{J^\pi}, \quad (5)$$

with the renormalized RPA matrices $\mathcal{A}$ and $\mathcal{B}$:

$$\begin{aligned} \mathcal{A}^{J^\pi}_{pn,p'n'} = {} & (E_p + E_n)\delta_{pp'}\delta_{nn'} \\ & - 2\big[g_{\mathrm{pp}} G(pn,p'n';J^\pi)(u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'}) \\ & + g_{\mathrm{ph}} F(pn,p'n';J^\pi)(u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'}) \big] \\ & \times \sqrt{D_{pn} D_{p'n'}}, \end{aligned} \quad (6)$$

$$\begin{aligned} \mathcal{B}^{J^\pi}_{pn,p'n'} = {} & 2\big[g_{\mathrm{pp}} G(pn,p'n';J^\pi)(u_p u_n v_{p'} v_{n'} + v_p v_n u_{p'} u_{n'}) \\ & - g_{\mathrm{ph}} F(pn,p'n';J^\pi)(u_p v_n v_{p'} u_{n'} + v_p u_n u_{p'} v_{n'}) \big] \\ & \times \sqrt{D_{pn} D_{p'n'}}. \end{aligned} \quad (7)$$

The particle-particle (G) and the particle-hole (F) matrix elements of the two-body nucleon-nucleon interaction [21] are scaled by the factors g_{pp} and g_{ph}, respectively, to account for the finite range of the nucleus and limited model space [2]. In the calculations we have chosen two values of the g_{ph} parameter (0.8 and 1.0) and leave g_{pp} as a free parameter of the theory [10,11]. E_p and E_n are the proton and neutron quasiparticle energies and the u's and v's are the usual BCS occupation factors.

The crucial point of the RQRPA is the calculation of the renormalization matrix D_{pn}. This can be achieved with the help of the mapping [12]

$$[a_p^\dagger \tilde{a}_p]_{00} \mapsto \hat{j}_p^{-1} \sum_{J^\pi M n} A^\dagger_{(pn)J^\pi M} A_{(pn)J^\pi M},$$
$$[a_n^\dagger \tilde{a}_n]_{00} \mapsto \hat{j}_n^{-1} \sum_{J^\pi M p} A^\dagger_{(pn)J^\pi M} A_{(pn)J^\pi M} \quad (8)$$

and inversion of (4). Equations (1)-(8) became coupled and can be solved by the iteration procedure we call "inner iteration": we start with $n_p = n_n = 0$, *i.e.* QRPA solution, calculate new quasiparticle densities and input them back again, till the convergence is achieved.

Now we proceed with the SRQRPA "outer iteration". This is necessary, since the RQRPA ground state has a non-vanishing quasiparticle content, while the BCS ground state is the quasiparticle vacuum. We relax the latter requirement and rewrite the BCS equations, by re-calculating the density matrix ρ and the pairing tensor κ:

$$\rho_a \equiv \left\langle 0 \left| c^\dagger_\alpha c_\alpha \right| 0 \right\rangle = v_a^2 + (u_a^2 - v_a^2) n_a, \quad (9)$$

$$\kappa_a \equiv \left\langle 0 \left| \tilde{c}_\alpha c_\alpha \right| 0 \right\rangle = u_a v_a (1 - 2n_a), \quad (10)$$

The u and v coefficients and quasiparticle energies are then obtained by minimizing the BCS ground-state energy. To solve the SRQRPA equations we start with the ordinary BCS equations, putting $n_p = n_n = 0$, then proceed with the corresponding RQRPA problem (inner iteration), that gives us new quasiparticle densities and loop with them back to BCS until the convergence is achieved (outer iteration).

3 Model parameters and results

In the calculations we used the same approach, as described in [17]. The two-body matrix elements were calculated from the Bonn-B nucleon-nucleon one-boson exchange potential [22] within the Brueckner theory [21]. The single-particle energies were calculated from the Coulomb-corrected Woods-Saxon potential with Bertsch parametrization [23]. As previously, we have found weak dependence of the RQRPA and the SRQRPA results on the dimension of the single-particle basis, the contrary to the QRPA behaviour. The conclusion is that the most suitable single-particle basis for all the nuclei in the mass range $100 < A \leq 150$ should contain 16 nlj shells (both for protons and neutrons) with ^{40}Ca as an inert core: $1p_{1/2}$, $1p_{3/2}$, $0f_{5/2}$, $0f_{7/2}$, $2s_{1/2}$, $1d_{3/2}$, $1d_{5/2}$, $0g_{7/2}$, $0g_{9/2}$, $2p_{1/2}$, $2p_{3/2}$, $1f_{5/2}$, $1f_{7/2}$, $0h_{9/2}$, $0h_{11/2}$, $0i_{13/2}$.

The calculations were performed for all the doubly-beta-decaying nuclei in the aforementioned mass range, except from ^{114,116}Cd and ^{122,124}Sn where the QRPA theory fails, due to the magic number of protons in the tin isotopes. To show the differences between the QRPA, the RQRPA and the SRQRPA and to illustrate much better the stability of the SRQRPA solutions we plot in figs. 1 to 10 the results of our calculations as a function of

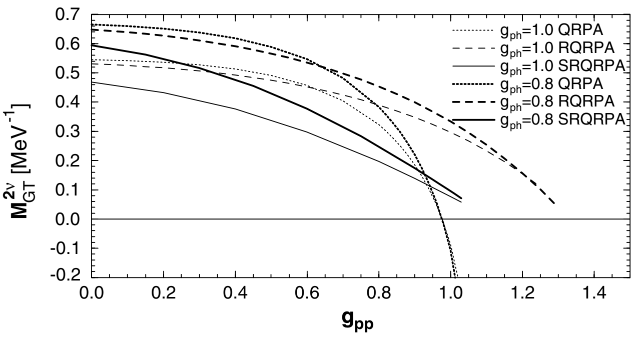

Fig. 1. The ^{104}Ru → ^{104}Pd $2\nu\beta\beta$ decay Gamow-Teller matrix element as a function of the g_{pp} parameter.

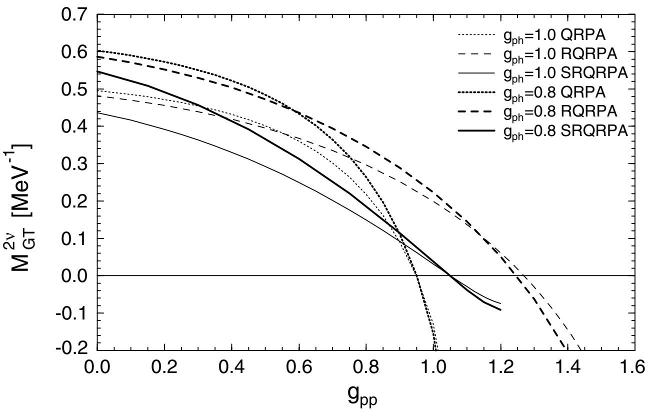

Fig. 2. Same as fig. 1, but for the ^{110}Pd → ^{110}Cd decay.

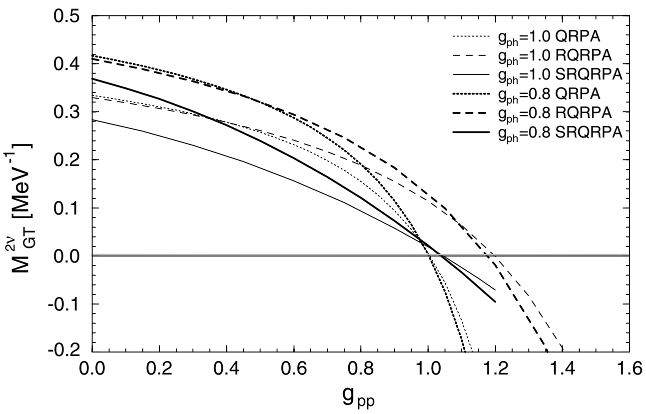

Fig. 3. Same as fig. 1, but for the ^{128}Te → ^{128}Xe decay. The shaded region shows the experimental data [24].

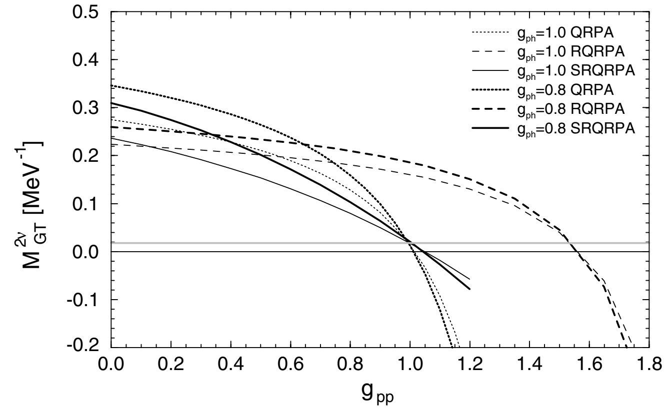

Fig. 4. Same as fig. 1, but for the ^{130}Te → ^{130}Xe decay. The shaded region shows the experimental data [24].

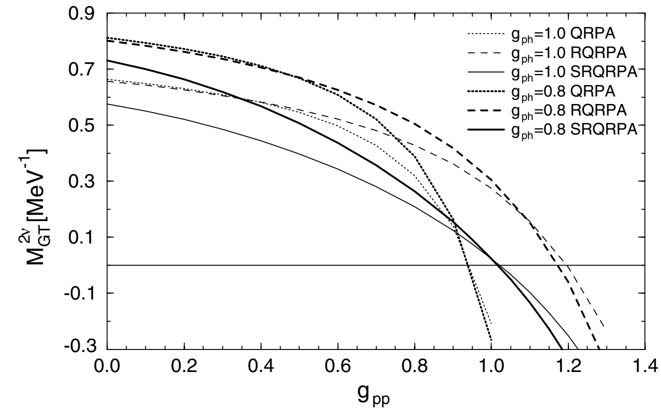

Fig. 5. Same as fig. 1, but for the ^{134}Xe → ^{134}Ba decay.

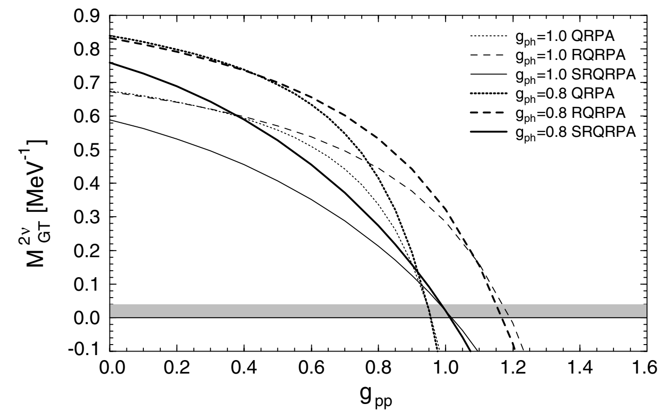

Fig. 6. Same as fig. 1, but for the ^{136}Xe → ^{136}Ba decay. The shaded region shows the experimental data [25].

the particle-particle (g_{pp}) factor. It can be seen that the RQRPA method tends to reproduce the existing experimental data for higher g_{pp} values (around 1.1 to 1.4), while the SRQRPA approach requires lower g_{pp} values (around 0.6 to 1.0) to fit the experiment.

The comparison between the QRPA, the RQRPA and the SRQRPA results in the considered range of the g_{pp} parameter shows the main features of the extended versions of the theory: the inclusion of the ground-state correlations beyond QRPA is not only improving the agreement between theoretical calculations and experimental data but also causes the stabilization of the dependence of $M^{2\nu}_{\rm GT}$ as a function of g_{pp}. Moreover, the iteration procedure for quasiparticle densities, which causes the treating RQRPA and BCS on the same footing, stabilizes the results even further. This behaviour can be explained by the suppression of ground-state correlations in the RQRPA and the SRQRPA methods. As a summary, in fig. 11 and table 1 we compare the range of results that can be obtained from

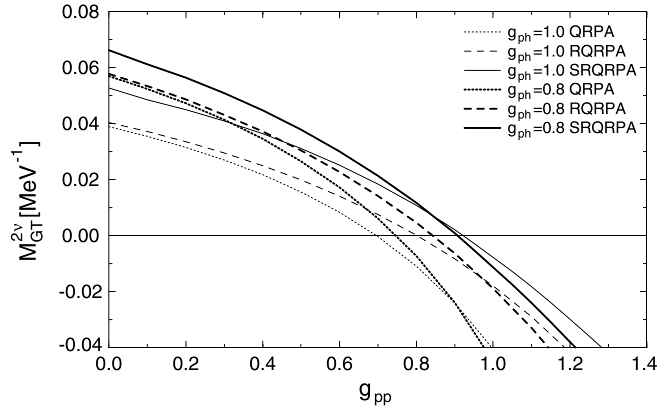

Fig. 7. Same as fig. 1, but for the $^{142}Ce \rightarrow {}^{142}Nd$ decay.

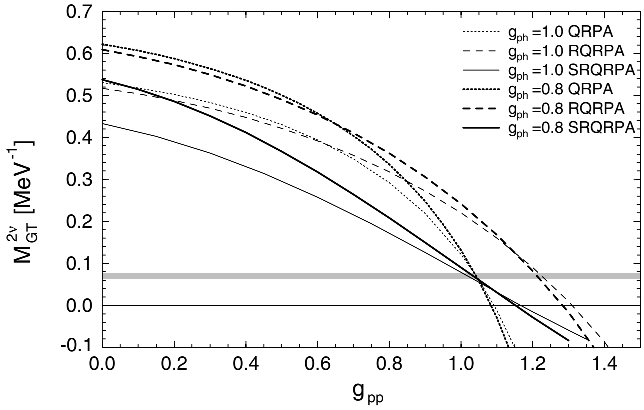

Fig. 8. Same as fig. 1, but for the $^{146}Nd \rightarrow {}^{146}Sm$ decay.

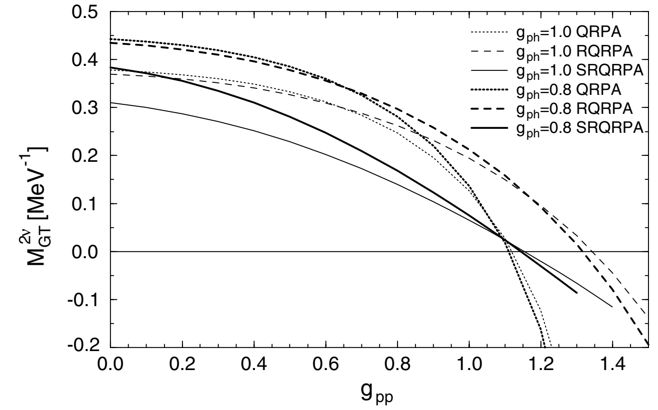

Fig. 9. Same as fig. 1, but for the $^{148}Nd \rightarrow {}^{148}Sm$ decay.

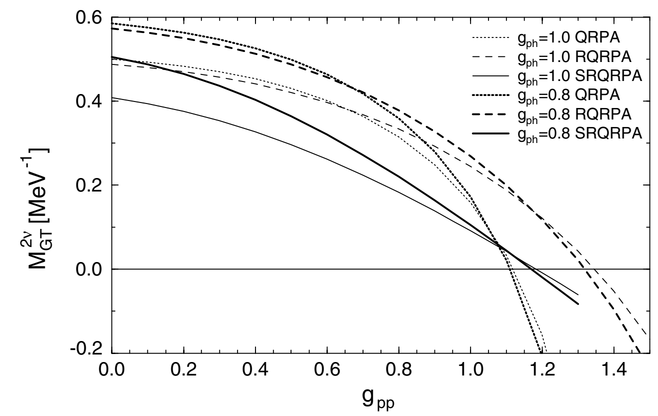

Fig. 10. Same as fig. 1, but for the $^{150}Nd \rightarrow {}^{150}Sm$ decay. The shaded region shows the experimental data [26].

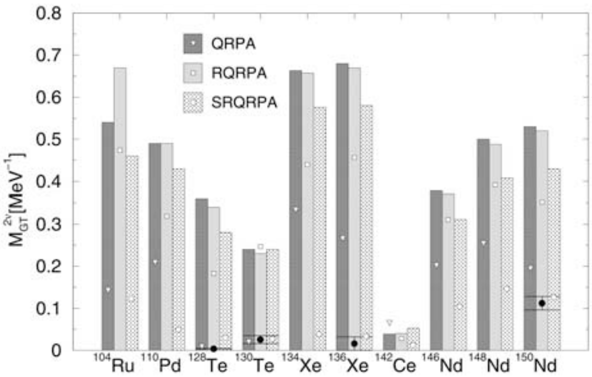

Fig. 11. Range of $M^{2\nu}_{\rm GT}$ values, calculated using three different QRPA approaches (vertical bars) and compared with the available experimental data (points with error bars). The open symbols show the calculated values for $g_{\rm ph} = g_{\rm pp} = 1.0$.

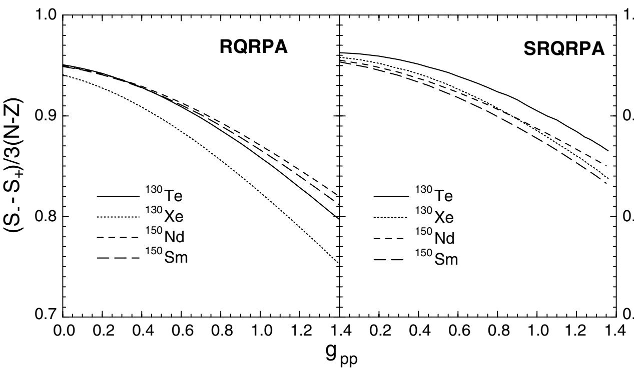

Fig. 12. Ikeda sum rule violation for RQRPA and SRQRPA as a function of $g_{\rm pp}$.

all three approaches and the available experimental data. One can see that both QRPA and the SRQRPA reproduces the experimental data quite nicely for $g_{\rm pp} \approx 1.0$, whereas the RQRPA fails and needs much higher (and rather unphysical) value of this parameter to get close to the experiment. This effect is probably due to the lack of internal consistency in the RQRPA approach [17].

It is well known that the ordinary QRPA method preserves the Ikeda sum rule, as long as all spin partners are taken into account. It means that the difference of β^- and β^+ transitions strengths in even-even nuclei is constant [15] and given by

$$S_I \equiv \sum_{m\mu} \langle 1^+_m \mu | \beta^-_\mu | 0^+_{\rm g.s.} \rangle^2 - \sum_{m\mu} \langle 1^+_m \mu | \beta^+_\mu | 0^+_{\rm g.s.} \rangle^2 = 3(N-Z), \tag{11}$$

where $\beta^\pm_\mu$ are the Gamow-Teller transition operators. However, the situation is different in renormalized versions

Table 1. Comparison of the available $2\nu\beta\beta$ experimental data with theoretical predictions obtained using three QRPA approaches for $g_{\rm ph} = 1.0$ and some chosen values of the $g_{\rm pp}$ parameter. The phase-space factors were taken from [27].

	$T_{1/2}^{2\nu\beta\beta}$			
$g_{\rm pp}$	0.6	0.8	1.0	1.2
^{104}Ru → ^{104}Pd	—			
QRPA	5.21e20	1.05e21	1.37e22	—
RQRPA	5.33e20	7.17e20	1.24e21	4.49e21
SRQRPA	1.23e21	2.80e21	1.84e22	—
^{110}Pd → ^{110}Cd	—			
QRPA	1.94e19	5.28e19	1.47e20	—
RQRPA	1.86e19	2.85e19	6.33e19	6.68e20
SRQRPA	4.03e19	1.13e20	2.59e21	4.57e20
^{128}Te → ^{128}Xe	(7.7 ± 0.4)e24 [24]			
QRPA	2.20e22	4.92e22	3.54e25	8.22e21
RQRPA	2.04e22	3.30e22	9.03e22	1.45e27
SRQRPA	4.82e22	1.37e23	3.38e24	2.34e23
^{130}Te → ^{130}Xe	(2.7 ± 0.1)e21 [24]			
QRPA	5.78e18	1.27e19	1.33e21	2.90e18
RQRPA	5.44e18	6.39e18	8.13e18	1.23e19
SRQRPA	1.21e19	3.30e19	7.09e20	6.26e19
^{134}Xe → ^{134}Ba	—			
QRPA	4.67e21	1.13e22	2.67e22	—
RQRPA	4.26e21	6.30e21	1.54e22	3.73e25
SRQRPA	9.82e21	2.66e22	1.98e24	1.84e22
^{136}Xe → ^{136}Ba	>3.6e20 [25], >5.5e20 [28]			
QRPA	7.94e17	1.82e18	7.52e18	—
RQRPA	7.15e17	1.05e18	2.54e18	7.95e20
SRQRPA	1.67e18	4.56e18	4.75e20	—
^{142}Ce → ^{142}Nd	—			
QRPA	1.97e23	1.16e23	8.35e21	1.64e21
RQRPA	6.90e22	3.06e26	4.31e22	7.92e21
SRQRPA	2.17e22	1.18e23	2.33e23	1.56e22
^{146}Nd → ^{146}Sm	—			
QRPA	2.11e30	3.38e30	1.30e31	1.38e31
RQRPA	2.15e30	2.99e30	5.47e30	2.21e31
SRQRPA	5.02e30	1.05e31	4.84e31	4.98e32
^{148}Nd → ^{148}Sm	—			
QRPA	5.79e18	9.46e18	3.72e19	3.85e19
RQRPA	5.94e18	8.41e18	1.56e19	6.30e19
SRQRPA	1.36e19	2.80e19	1.12e20	1.30e22
^{150}Nd → ^{150}Sm	$(6.75^{+0.37}_{-0.42} \pm 0.68)$e18 [26], $(1.88^{+0.66}_{-0.39} \pm 0.19)$e19 [29]			
QRPA	5.41e16	9.84e16	5.70e17	1.98e17
RQRPA	5.48e16	8.32e16	1.73e17	1.03e18
SRQRPA	1.27e17	2.97e17	1.36e18	3.76e19

of QRPA. Since the ground state in RQRPA and SRQRPA is not a pure BCS state, cancellation of the so-called scattering terms does not occur, giving rise to Ikeda sum rule violation (fig. 12). As a general rule, the violation is less significant in heavy nuclei and grows with $g_{\rm pp}$. It depends also on the number of J^π multipolarities used in calculations. More exact calculations, with bigger number of multipolarities taken into account, show clearly that the sum rule is not fulfilled. Here, we repeat our conclusions from ref. [30] that SRQRPA violates the Ikeda sum rule to significantly smaller degree than the RQRPA.

4 Conclusions

We have calculated the double Gamow-Teller nuclear matrix elements for the two-neutrino double-beta-decay to the ground state of the nuclei with $100 < A \leq 150$. We have compared the QRPA approach to its extensions, that take the Pauli inclusion principle into account, *i.e.* the RQRPA and the new self-iterative BCS + RQRPA method (SRQRPA). Unlike the old QRPA which requires fine tuning of the particle-particle strength parameter, the RQRPA and SRQRPA methods give stable matrix elements over the whole range of $g_{\rm pp}$ and thus allow for more predictive power. We have shown that inclusion of the ground-state correlations beyond QRPA causes some stabilization of the Gamow-Teller matrix elements, because of the additional change of the quasiparticle densities during the iteration procedure with the modified BCS solution.

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